

Compound Gear Train Dynamics

| LAB OBJECTIVES

The purpose of this experiment is to combine rotational dynamics and gear train theory to experimentally determine the mechanical efficiency of a compound gear train driven by a hanging mass. Students will first characterize the system inertia independently, then design and execute their own investigation to connect gear train kinematics, Newton's second law for rotation, power transmission, and real-world losses.

| EDUCATIONAL OBJECTIVES

- Experimentally determine the moment of inertia of the flywheel using a hanging mass and PASCO rotary sensor.
- Predict angular acceleration of each shaft using reflected inertia and Newton's second law.
- Measure angular acceleration experimentally and compare to theoretical predictions.
- Calculate input and output torque and power from sensor data.
- Determine the mechanical efficiency of the gear train and identify sources of loss.
- Communicate experimental methods, results, and conclusions clearly and professionally.

| EQUIPMENT

- Compound gear train frame (3-shaft, 4-gear increaser: 24T–24T–64T–18T)
- Flywheel (removable, output shaft)
- Input shaft pulley with string
- Hanging masses: 50 g, 100 g, 200 g
- 2× PASCO rotary motion sensors (ME-8951) with Bluetooth transmitters
- PASCO Capstone software
- Calipers, scale, Allen wrench

| BACKGROUND

GEAR TRAIN KINEMATICS

A compound gear train transmits motion between an input shaft and an output shaft through intermediate gears. The overall speed ratio e is the product of each stage's individual ratio, where each stage ratio is the number of teeth on the driver divided by the number of teeth on the driven gear:

$$e = (N_1/N_2) \times (N_3/N_4) = \omega_{\text{output}} / \omega_{\text{input}}$$

For this gear train (24T → 24T → 64T → 18T), the output shaft spins faster than the input ($e > 1$). Speed and torque are inversely related — the gear train trades torque for speed.

NEWTON'S SECOND LAW FOR ROTATION

For a rigid body constrained to rotate about a fixed axis, the net torque equals the mass moment of inertia times the angular acceleration:

$$\Sigma \mathbf{M}_O = I_O \cdot \alpha$$

When multiple rotating components are connected through gears, all inertias must be reflected (referred) to a single shaft before applying this equation. The reflection rule to the output shaft is:

$$I_{\text{total}} = I_{\text{output}} + I_{\text{middle}} \cdot (1/e)^2 + I_{\text{input}} \cdot (1/e)^2$$

Note that inertia on a faster-spinning shaft (input) is divided by e^2 when referred to a slower output shaft — it contributes less. The reverse is true when the output is faster, as in this gear increaser.

EQUATION OF MOTION — HANGING MASS SYSTEM

For a hanging mass m driving the input shaft through a pulley of radius r , the string tension is:

$$T = m(g - a)$$

where a is the linear acceleration of the mass. Applying Newton's second law and the kinematic constraint $a = \alpha_{\text{input}} \cdot r$, and reflecting everything to the input shaft:

$$\alpha_{\text{input}} = (m \cdot g \cdot r) / (I_{\text{total}}/e^2 + m \cdot r^2)$$

$$a = \alpha_{\text{input}} \cdot r$$

$$\alpha_{\text{output}} = \alpha_{\text{input}} \cdot e$$

POWER AND MECHANICAL EFFICIENCY

Power transmitted at any shaft is torque times angular velocity:

$$P = \tau \cdot \omega$$

Mechanical efficiency compares output power to input power. In an ideal (frictionless) system $\eta = 1$. Real systems lose power to bearing friction and gear mesh losses:

$$\eta = P_{\text{output}} / P_{\text{input}} = (\tau_{\text{output}} \cdot \omega_{\text{output}}) / (\tau_{\text{input}} \cdot \omega_{\text{input}})$$

Torque at the input shaft comes from the string tension: $\tau_{\text{input}} = T \cdot r_{\text{pulley}}$. Torque at the output shaft can be found from Newton's second law using only the output shaft's own inertia (not the full reflected system):

$$\tau_{\text{output}} = I_{\text{output_only}} \cdot \alpha_{\text{output}}$$

| PART A — FLYWHEEL CHARACTERIZATION (GUIDED)

Before engaging the gear train, you will determine the flywheel's moment of inertia experimentally. This value will be provided to you as known data for all subsequent calculations.

 The flywheel sits on the output shaft **ONLY** for this part. All gears are disengaged. This isolates the flywheel + output shaft + sensor as the only rotating system.

SETUP

- Mount the flywheel on the output shaft with the gears disengaged.
- Wrap the string around the output shaft pulley and attach a hanging mass.
- Connect the PASCO sensor on the output shaft to Capstone via Bluetooth.
- In Capstone: create a graph of angular velocity (ω) vs. time. Set sample rate to 50 Hz.

PROCEDURE

1. Measure and record the output shaft pulley radius r_{out} using calipers.
2. Weigh the hanging mass and record it as m_{hang} .
3. Hold the mass at its highest point, press Record in Capstone, then release.
4. Let the mass fall and stop recording when it reaches the bottom.
5. Use the linear fit tool in Capstone to find the slope of the ω vs. t curve. This slope is α_{output} (rad/s^2).
6. Repeat 3 times and record each α value. Compute the mean and standard deviation.

CALCULATIONS

From your measured α_{output} , back-calculate the moment of inertia of the flywheel + output shaft + sensor assembly. Starting from the equation of motion for this isolated system:

$$m \cdot g \cdot r = (I_{\text{system}} + m \cdot r^2) \cdot \alpha_{\text{output}}$$

$$I_{\text{system}} = (m \cdot g \cdot r / \alpha_{\text{output}}) - m \cdot r^2$$

This I_{system} is the experimentally determined moment of inertia. You will use this as your baseline inertia for the output shaft in all subsequent parts.

Trial	Hanging Mass (kg)	Pulley Radius (m)	α_{output} (rad/s ²)	I_{system} (kg·m ²)
1				
2				
3				
Mean $\pm$ Std				

| PART B — STUDENT-LED INVESTIGATION

Using the flywheel inertia you measured in Part A as your foundation, you will now design and execute your own experimental investigation of the full gear train. Your goal is to experimentally determine the mechanical efficiency of the gear train and understand what causes losses.

 *You are responsible for designing your procedure. The steps below are a framework — not a complete recipe. Your team must decide what to measure, what to calculate first, and in what order. Justify your choices in your post-lab writeup.*

REQUIRED OUTCOMES

Your investigation must produce all of the following. The order in which you arrive at them is up to you:

- Theoretical prediction of α_{input} and α_{output} using reflected inertia and the equation of motion.
- Experimental measurement of α_{input} and α_{output} from both PASCO sensors simultaneously.
- Experimental verification of the gear speed ratio from the two sensor traces.
- Input torque τ_{input} calculated from string tension and pulley radius.
- Output torque τ_{output} calculated from Newton's second law using output shaft inertia only.
- Input power P_{input} and output power P_{output} at a defined moment in time during the run.
- Mechanical efficiency $\eta = P_{\text{output}} / P_{\text{input}}$.
- Comparison of ideal torque ratio (from gear teeth) vs. experimental torque ratio (from your measured values).
- Identification and discussion of at least two sources of mechanical loss in the system.

SUGGESTED STARTING QUESTIONS

If you are unsure where to begin, consider these:

- *What do you need to know before you can predict α ? What quantities do you already have from Part A?*
- *How does the reflected inertia change now that the gear train is engaged? Which shaft's inertia dominates?*
- *At what point in time during the run will you evaluate power — and why does that choice matter?*
- *How will you extract torque from sensor data without a load cell?*

PASCO CAPSTONE SETUP FOR FULL GEAR TRAIN RUN

7. Connect both rotary sensors (input shaft and output shaft) to Capstone.
8. Create a single graph with TWO ω vs. t traces — one per sensor — overlaid on the same axes.
9. Set sample rate to 50 Hz on both sensors. Check sign convention: both sensors should show positive ω during a forward run. Flip the sign in sensor settings if needed.
10. Hold the hanging mass at its highest point, press Record, release, and stop recording when the mass reaches the bottom.

11. Use the linear fit tool on each trace separately to extract α_{input} and α_{output} from the slopes.

12. Repeat at least 3 times and report mean and standard deviation for each α .

Trial	α_{input} (rad/s ²)	α_{output} (rad/s ²)	$e_{\text{exp}} = \alpha_{\text{out}}/\alpha_{\text{in}}$	Note
1				
2				
3				
Mean $\pm$ Std				

EFFICIENCY CALCULATION SUMMARY TABLE

Complete the following table using your experimental data and calculations. Show all work in your post-lab writeup.

Quantity	Symbol	How Obtained	Value
Input pulley radius	r_{in}	Caliper measurement	
String tension	$T = m(g-a)$	From measured a	
Input torque	$\tau_{\text{in}} = T \cdot r_{\text{in}}$	Calculation	
Input ang. velocity (at t^*)	ω_{in}	Pasco sensor (input)	
Input power	$P_{\text{in}} = \tau_{\text{in}} \cdot \omega_{\text{in}}$	Calculation	
Output shaft inertia	I_{out}	From Part A	
Output torque	$\tau_{\text{out}} = I_{\text{out}} \cdot \alpha_{\text{out}}$	Newton's 2nd law	
Output ang. velocity (at t^*)	ω_{out}	Pasco sensor (output)	
Output power	$P_{\text{out}} = \tau_{\text{out}} \cdot \omega_{\text{out}}$	Calculation	
Mechanical efficiency	$\eta = P_{\text{out}} / P_{\text{in}}$	Calculation	
Ideal torque ratio	$\tau_{\text{in}}/\tau_{\text{out ideal}} = e$	From gear teeth	
Experimental torque ratio	$\tau_{\text{in}}/\tau_{\text{out exp}}$	From measured torques	

* Choose a consistent time t^* during the linear (constant acceleration) portion of the run — the same t^* for both shafts.

| OPTIONAL EXTENSION — FLYWHEEL LOCATION INVESTIGATION

If time permits, move the flywheel from the output shaft to the input shaft and repeat your full experiment. Before running:

- Predict the new α_{output} . How does moving the flywheel change the reflected inertia? Show your calculation.
- Predict the new efficiency. Do you expect it to be higher, lower, or the same? Why?

Then run the experiment and compare your predictions to the measurements. What does this tell you about the relationship between inertia location and system dynamics?

| POST-LAB DELIVERABLES

Submit a single PDF on Canvas. All work must meet professional standards: legible handwriting or typed equations, neat sketches drawn with a straight edge, proper units on every quantity.

PART A — FLYWHEEL CHARACTERIZATION

- Completed data table with all three trials, mean, and standard deviation. (3 pts)
- Calculation of I_{system} with all steps shown. (4 pts)

PART B — GEAR TRAIN INVESTIGATION

- Documented procedure: what you chose to measure and in what order, with justification. (4 pts)
- All theoretical predictions (α_{input} , α_{output} , torque ratio) with full derivations. (6 pts)
- Completed raw data tables for angular acceleration trials. (4 pts)
- Completed efficiency summary table with all supporting calculations. (6 pts)
- Comparison of ideal vs. experimental torque ratio with percent error. (3 pts)
- Comparison of ideal vs. experimental gear speed ratio with percent error. (3 pts)

DISCUSSION

- One paragraph: quantitative comparison of experimental vs. predicted results. What agrees well? What doesn't? (4 pts)
- One paragraph: identify at least two physical sources of loss and explain how each one reduces efficiency. (4 pts)
- One paragraph: if you repeated this experiment with a different hanging mass, how would efficiency change and why? (3 pts)

Only one team member needs to submit. Ensure all participating team members' names appear on the first page.